

## Exercise 18

Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function.

$$y = \int_{\sin x}^1 \sqrt{1+t^2} dt$$

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### Solution

According to part 1 of the fundamental theorem of calculus,

$$\frac{d}{dx} \int_a^x f(t) dt = f(x).$$

Switch the limits of integration to put the variable part on top.

$$y = - \int_1^{\sin x} \sqrt{1+t^2} dt$$

In order to make the upper limit a single variable, let  $u = \sin x$ .

$$y = - \int_1^u \sqrt{1+t^2} dt$$

As a result, using the chain rule,

$$\begin{aligned} y' &= - \frac{d}{dx} \int_1^u \sqrt{1+t^2} dt \\ &= - \frac{du}{dx} \frac{d}{du} \int_1^u \sqrt{1+t^2} dt \\ &= - \frac{du}{dx} \left( \sqrt{1+u^2} \right) \\ &= -(\cos x) \sqrt{1+\sin^2 x}. \end{aligned}$$